Diagrammatic Extension of Konar's (2009) "Ray Measure of Point Price Elasticity of Demand"

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Abstract

The paper aimed to offer the diagrammatic extension of Konar's (2009) ray measure of point price elasticity of demand. "Ray" refers to the straight line drawn from the origin to a point on the rehashed demand curve. The ray measure indicates that the "absolute point price elasticity of demand" is equal to "the slope of the ray".

Keywords: **rehash, curvature, normal, supernormal**

JEL Classification: D10, D11,D19

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The objective of this article is to offer the diagrammatic extension of Konar's (2009) ray measure of "point price elasticity of demand" (*PPED*). "Ray" refers to the straight line drawn from the origin to a point on the "rehashed demand curve" (*RDC*) in the *xy*- space, where $x = |dP/dD| \equiv$ absolute marginal demand and $y = (P/D) \equiv$ average demand for the negatively sloping demand curve (*DC*), denoted by $P = P(D)$ *such that* $P'(D) < 0$. The *RDC* is the mapping of the *DC* from *DP*-space into *xy*-space, in which *x* and *y* are measured along the horizontal and vertical axes respectively, while in *DP*-space, *D* and *P* are also measured along the horizontal and vertical axes respectively. The *RDC* is denoted by $y = f(x)$, while the "marginal *RDC"* (*MRDC*) and the "average *RDC"* (*ARDC*) are respectively denoted by *dy/dx* and *y/x* (Konar, 2009). The ray measure indicates that the "absolute point price elasticity of demand" (*APPED*) = $|E_{DP}| = |(dD/D)/(dP/P)| = (P/D)/|(dP/dD)|$ = [Average Demand]/[Absolute Marginal Demand] = *y/x* = slope of the ray drawn from the origin to a given point on the *RDC* in the *xy*-space = *ARDC,* where $|E_{DP}| \equiv$ absolute point price (*P*) elasticity of demand (*D*). The significance of nomenclature of the ray measure is that only the ray in the *xy*- space determines the $|E_{DP}|$. The ray measure has been designed to disclose the true relationship between the *APPED* and the *D* along the ordinary (or normal) and extraordinary (supernormal) demand curves, which may be convex, concave, or negatively sloping linear. Moreover, the ray measure rules out the inadequacy of the length-ratio measure of Marshall's (1890) elasticity of demand.

Rehashed Demand Curve *(RDC)*

In order to know the features of the *RDC*, the following functions/curves should be considered:

Marginal Rehashed Demand Function (*MRDF*) $\equiv dy/dx = f'(x) =$

$$[P/D^2(1 + 1/|E_{DP}|)] / [d^2P/dD^2] \qquad (1)$$

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$$\frac{\partial\,(MRDF)}{\partial\,(d^2P/dD^2)} = -\frac{P/D^2(1 + 1/|E_{DP}|)}{(d^2P/dD^2)^2} < 0 \qquad (2)$$

$$\frac{\partial^2(MRDF)}{\partial\,(d^2P/dD^2)^2} = \frac{P/D^2(1 + 1/|E_{DP}|)}{(d^2P/dD^2)^3} > 0 \qquad (3)$$

$$ARDC \equiv y/x = |E_{DP}| = \text{slope of the ray in the } xy\text{ - space} \qquad (4)$$

Elasticity of y with respect to x along the RDC ≡

$$E_{yx} = \frac{dy/y}{dx/x} = \frac{dy/dx}{y/x} = \frac{MRDF}{ARDF} = \frac{MRDF}{|E_{DP}|} = \frac{P/D^2[(1+|E_{DP}|)/(|E_{DP}|)^2]}{(d^2P/dD^2)} \qquad (5)$$

$$\frac{\partial E_{yx}}{\partial(d^2P/dD^2)} = -\frac{P/D^2\,[(1 + |E_{DP}|)/(|E_{DP}|)^2]}{(d^2P/dD^2)^2} \qquad (6)$$

$$\frac{\partial^2 E_{yx}}{\partial(d^2P/dD^2)^2} = \frac{P/D^2\,[(1 + |E_{DP}|)/(|E_{DP}|)^2]}{(d^2P/dD^2)^3} \qquad (7)$$

$$\frac{\partial E_{yx}}{\partial(|E_{DP}|)} = -\frac{dy/dx}{(|E_{DP}|)^2} = -\frac{MRDF}{(|E_{DP}|)^2} \qquad (8)$$

Equations (2) and (3) indicate that the *MRDC* is inversely related to the curvature of the *DC* (i.e. d^2P/dD^2), as shown in Figure 1 and Figure 2. From Figure 2 and Figure 3, we find that the *RDCs* denoted by $y = f_i(x)$ [i = 1, 2, 3] in Figure 3, are the mapping of *DCs,* denoted by $P = P_i(D)$ [i = 1, 2, 3] in Figure 2, from *DP*- space into *xy*-space.

From equations (2) and (3), we get the following results:

(1) If the *DC* is linear (i.e. $d^2P/dD^2 = 0$), the *RDC* is infinitely sloping (i.e. $dy/dx = \infty$).

(2) If the *DC* is convex (i.e. $d^2P/dD^2 > 0$), the *RDC* is positively sloping (i.e. $dy/dx > 0$) and higher or lower convexity of the *DC* gives rise to flatter or steeper positively sloping *RDC*.

(3) If the *DC* is concave to the origin (i.e. $d^2P/dD^2 < 0$), the *RDC* is negatively sloping (i.e. $dy/dx < 0$) and higher or lower concavity of the *DC* implies flatter or steeper negatively sloping *RDC*.

Figure 1. Set of *DCs*

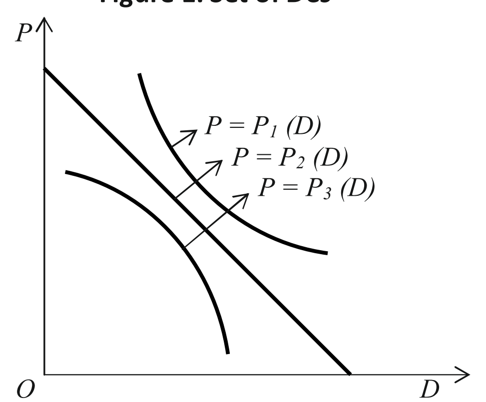


Figure 2. Set of *RDCs*

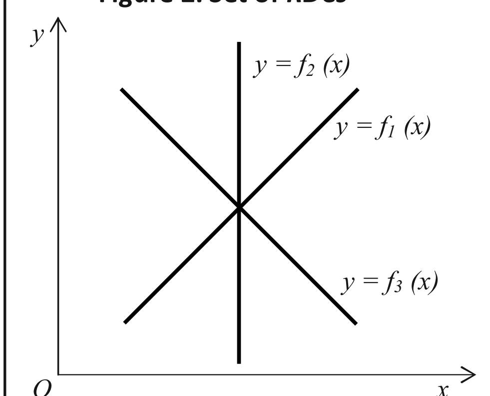
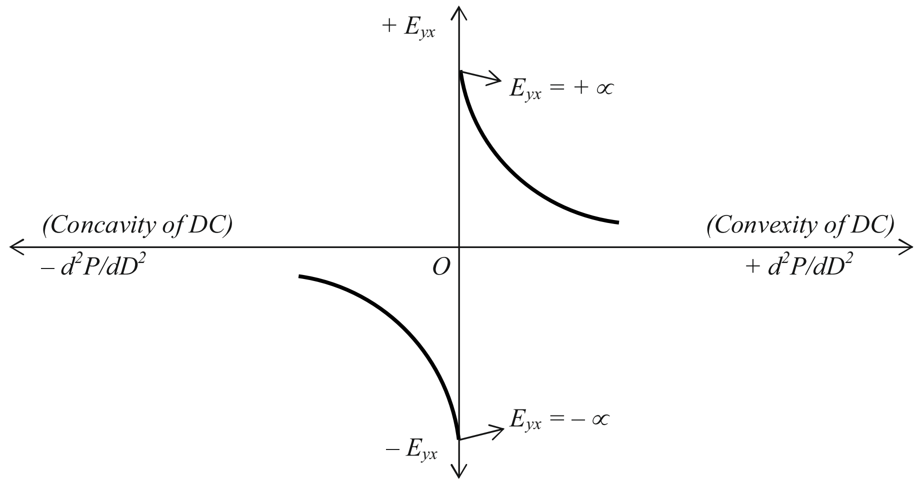


Figure 3. Relation Between the Curvature of *DC* and the Elasticity of *RDC*



Further, from equations (5), (6) , and (7), we get the following results:

(1) E_{yx} is inversely related to the curvature of the *DC* (i.e. d^2P/dD^2).

(2) For linear *DC*, $E_{yx} = \propto$; for convex *DC*, $E_{yx} > 0$ and for concave *DC*, $E_{yx} < 0$.

(3) For convex *DC*, higher or lower curvature of the *DC* gives rise to the lower or higher $E_{yx} > 0$ of the positively sloping *RDC*.

(4) For concave *DC*, higher or lower curvature of the *DC* implies the higher or lower $E_{yx} < 0$, or alternatively lower or higher $|E_{yx}|$ of the negatively sloping *RDC*.

The relationship between the curvature of the *DC* and the E_{yx} of the *RDC* is shown in the Figure 3.

Mathematical Relationship between *APPED* and *D*

For the demand function $P = P(D)$, such that $P'(D) < 0$, the $APPED = |E_{DP}| = |(dD/D)/(dP/P)| = (P/D)/|(dP/dD)|$
$= (P/D)/(-dP/dD) = (y/x) =$ slope of the ray in the *xy* - space = *ARDC* (9)

$$Now,\ d(|E_{DP}|)/dD = d[-(P/D)/(dP/dD)]dD = -d[(P/D)/(dP/dD)]dD$$
$$= [DP''(D) + 2P'(D)] - \{P'(D)/P(D)\}[DP'(D) + P(D)] \quad (10)$$
$$= P(D)/D[P''(D) - (P/D^2)\{(1 - |E_{DP}|)/(|E_{DP}|)^2\}]/\{P'(D)\}^2 \quad (11)$$

In equations (10) and (11), *MR* (Marginal Revenue) = $[DP'(D) + P(D)]$, Slope of MR = $[DP''(D) + 2P'(D)]$, $P'(D)$ = Marginal Demand and $P(D)/D$ = Average Demand.

From equation (10), we get the following results:

(1) If the *DC* is concave to the origin, which means that $P = P(D)$ such that $P'(D) < 0$ *and* $P''(D) < 0$, we have $d(|E_{DP}|)/dD < 0$.

(2) If the *DC* is convex to the origin, which means that $P = P(D)$ such that $P'(D) < 0$ *and* $P''(D) > 0$, we have $d(|E_{DP}|)/dD > 0, = 0,$ or < 0.

(3) If the *DC* is negatively sloping linear, which means that $P = P(D)$ such that $P'(D) < 0$ *and* $P''(D) = 0$, we have $d(|E_{DP}|)/dD < 0$.

Thus, we find that all forms of concave and linear *DC* support the $d(|E_{DP}|)/dD < 0$, while this support is violated by some forms of convex *DC*. But these results are not final and they should be treated as "surface results". The "inner final results," most of which contradict the "surface results,"can be obtained from equation (11) as follows:

(a) If the *DC* is concave to the origin, which means that $P = P(D)$ such that $P'(D) < 0$ and $P''(D) < 0$, but if [exercise equation (11)] :

(a_1) $|E_{DP}| = 1$, we have $d(|E_{DP}|)/dD < 0$.
(a_2) $|E_{DP}| > 1$, we have $d(|E_{DP}|)/dD > 0, = 0$, or < 0.
(a_3) $|E_{DP}| < 1$, we have $d(|E_{DP}|)/dD < 0$.

Thus, from (a), we may conclude that most of the concave *DCs* obey the negative relationship between *APPED* and *D*.

(b) If the *DC* is convex to the origin, which means that $P = P(D)$ such that $P'(D) < 0$ *and* $P''(D) > 0$, but if [exercise equation (11)] :

(b_1) $|E_{DP}| = 1$, we have $d(|E_{DP}|)/dD > 0$.
(b_2) $|E_{DP}| > 1$, we have $d(|E_{DP}|)/dD > 0$.
(b_3) $|E_{DP}| < 1$, we have $d(|E_{DP}|)/dD > 0, = 0$ or, < 0.

Thus, from (b), we may conclude that almost all forms of convex *DC* do not obey the negative relationship between *APPED* and *D.*

(c) If the *DC* is negatively sloping linear, which means that $P = P(D)$ such that $P'(D) < 0$ and $P''(D) = 0$, but if [exercise equation (11)] :

(c_1) $|E_{DP}| = 1$, we have $d(|E_{DP}|)/dD = 0$.
(c_2) $|E_{DP}| > 1$, we have $d(|E_{DP}|)/dD > 0$.
(c_3) $|E_{DP}| < 1$, we have $d(|E_{DP}|)/dD < 0$.

Thus, from (c), we may conclude that very few forms of linear *DC* obey the negative relationship between *APPED* and *D.* Now, we can argue that the forms of *DC,* which do not support such traditionally established negative relationship between *APPED* and *D* are abnormal, supernormal, or extraordinary.

Diagrammatization of the Mathematical Relationship between *APPED* and *D*

Now, let us examine the relationship between *APPED* and *D* diagrammatically for the following *DCs* : (a) Linear *DC*, (b) Convex *DC,* and (c) Concave *DC*.

➪ **Linear DC :** Figure 4 and Figure 5 show that the linear *DC* in Figure 4 is mapped into the vertical *RDC* in Figure 5. The slope of the ray *OA'*, *OB'*, or *ON'* determines the $|E_{DP}|$ at the point *A* (or *A*'), *B* (or *B')*, or *N* (or *N'*). It is noteworthy that $E_{yx} = \propto$ for the vertical *RDC* in Figure 5 is consistent with the $\propto \geq |E_{DP}| \geq 0$ for the linear *DC* in Figure 4. Thus, we may conclude that this form of linear *DC* or vertical *RDC* [see (c_3) along with Figure 4 and Figure 5] fully supports the negative relationship between *APPED* and *D*, that is, $d(|E_{DP}|)/dD < 0$. But it does not imply that other forms of linear *DC* will support such negative relationship [see (c_1) and (c_2)].

➪ **Convex *DCs* :** From Figure 6, Figure 7, and Figure 8, we get the following results:

Figure 4. Linear *DC* with Different $|E_{DP}|$ **Figure 5. Vertical *DDC* with Different $|E_{DP}|$**

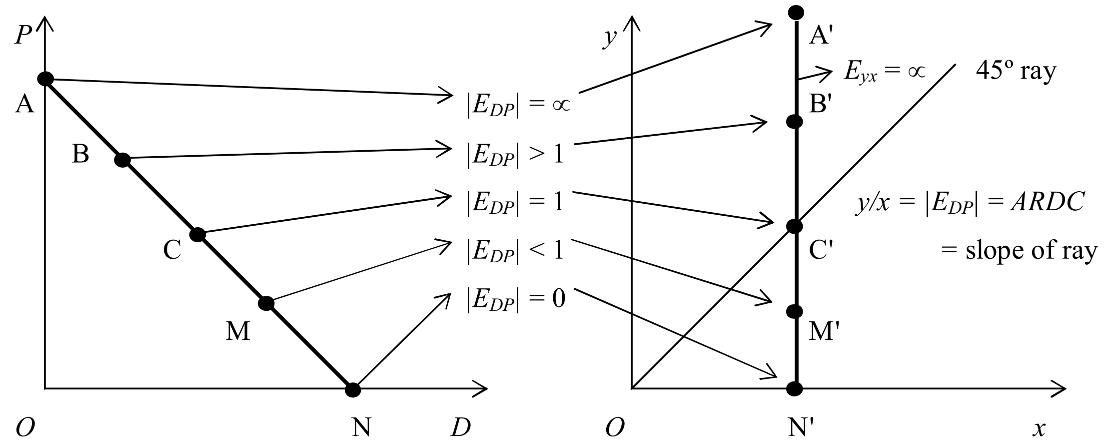


Figure 6. Convex *DC*s with Varying Degree of Convexity

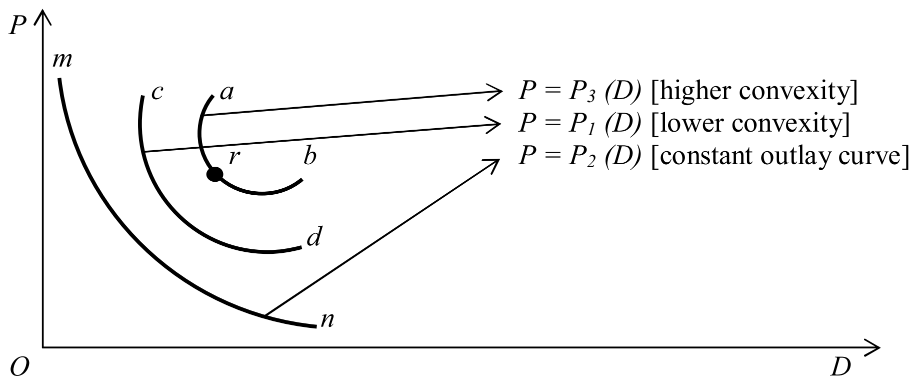
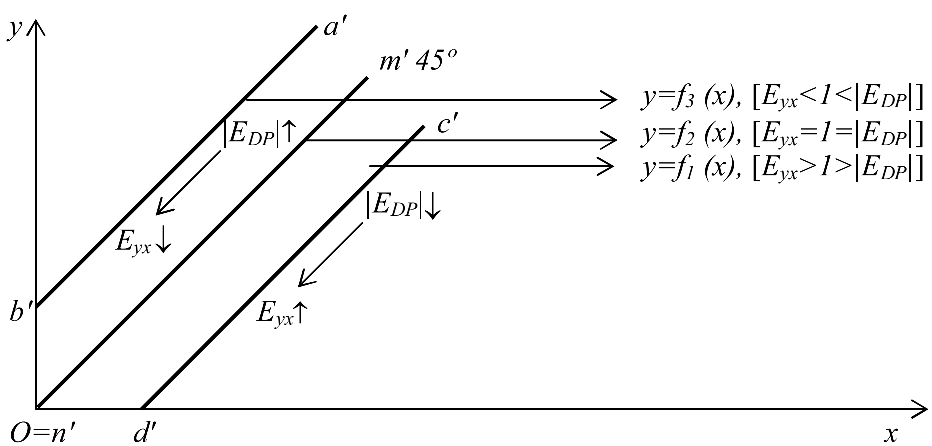


Figure 7. Linear *RDCs* with the Same Slope



(1) The *RDC* denoted by $y = f_i(x)$ [i = 1, 2, 3] in Figure 7 and Figure 8 is the mapping of *DC* denoted by $P = P_i(D)$ [i = 1, 2, 3] in Figure 6 from *DP*-space into *xy* - space.

(2) The slope of the ray in *xy* - space = $y/x = |E_{DP}| = ARDC$.

(3) Under the *ceteris paribus* assumption, the slope of *RDC* (i.e. dy/dx) is inversely related to the curvature of the *DC* (i.e. d^2P/dD^2).

(4) Under the *ceteris paribus* assumption, the E_{yx} of *RDC* is also inversely related to the curvature of *DC* (i.e. d^2P/dD^2).

(5) The positivity of E_{yx} is inversely related to the $|E_{DP}|$ excepting the rectangular hyperbolic *DC*.

(6) The foregoing negative relationship [i.e. $d(|E_{DP}|)/dD < 0$] holds only for the *RDCs*, which are denoted by *d'c'* in Figure 7 and *d'c'* in Figure 8. All other *RDCs* in Figure 7 and Figure 8 show either $d(|E_{DP}|)/dD > 0$, or $d(|E_{DP}|)/dD = 0$ [see (b_3)]. For example, in Figure 7, along the entire *RDC* denoted by *b'a'*, it is obvious that $d(|E_{DP}|)/dD > 0$ [see (b_2)]. Furthermore, in Figure 8, the *RDC* denoted by *b'a'* shows that along the entire *RDC*, $d(|E_{DP}|)/dD > 0$ [see (b_2) and (b_3]. Moreover, in both Figure 7 and Figure 8, the *RDC* denoted by *m'n'* shows that $d(|E_{DP}|)/dD = 0$.

Thus, mere convexity of the *DC* does not necessarily support the $d(|E_{DP}|)/dD < 0$. Whether a convex *DC* will support such a negative relationship depends upon the degree of convexity of the *DC*. What is more interesting is that mere $|E_{DP}| = 1$ does not necessarily imply $d(|E_{DP}|)/dD = 0$ for the convex *DC* [see (b_1)].

Figure 8. Linear *RDCs* with Different Slopes

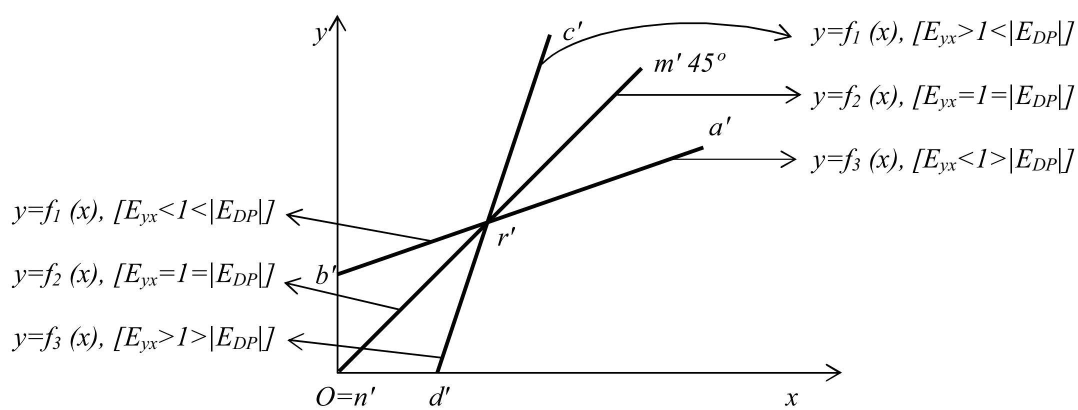


Figure 9. Set of Concave *DCs*

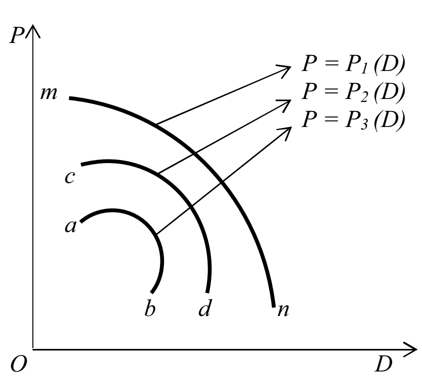


Figure 10. Set of Negatively Sloping *RDCs*

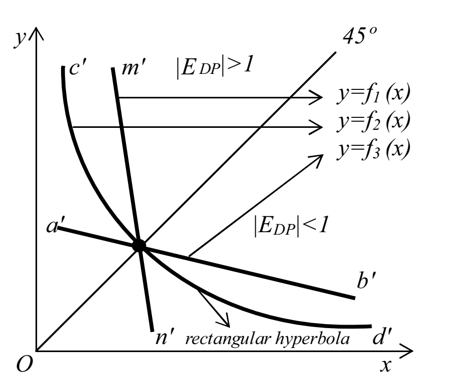
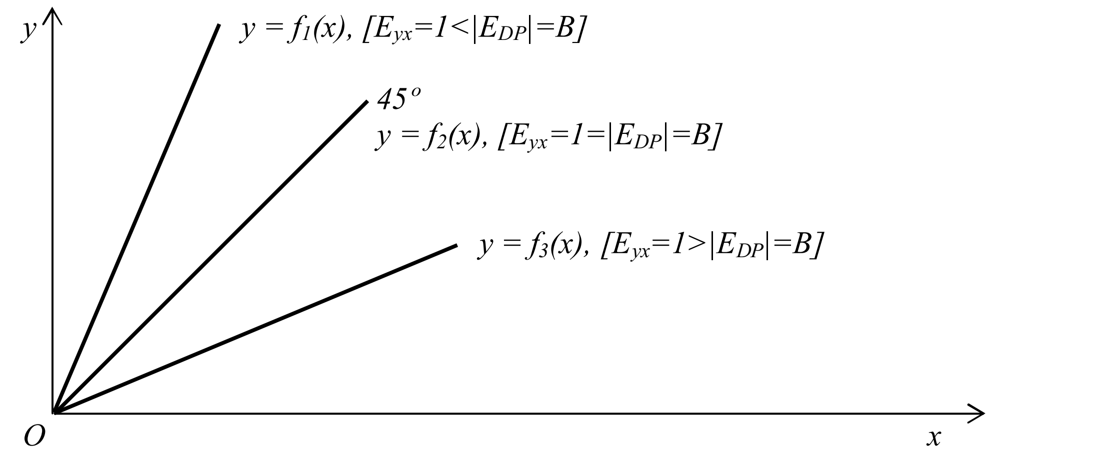


Figure 11. Linear *RDCs* Derived from Cobb-Douglas Type of Constant Outlay Curves



(7) If the *RDCs* in Figure 7 and Figure 8 become linear, or curvilinear with different slopes, then also the derived results will remain the same.

➯ **Concave *DCs* :** We shall now show that $d(|E_{DP}|)/dD < 0$ is fully supported by such forms of concave *DC,* which are displayed in Figure 10. This means that the falsification of such negative relationship by these forms of concave *DC* is absolutely ruled out.

The *RDC* denoted by $y = f_i(x)$ [i = 1, 2, 3] in Figure 10 is the mapping of *DC* denoted by $P = P_i(D)$ [i = 1, 2, 3] in Figure 9 from *DP* -space into *xy* -space. The 45º line in Figure 10 is treated as the dividing line of the whole *xy* -space into elastic zone (i.e. $|E_{DP}| > 1$) and inelastic zone (i.e. $|E_{DP}| < 1$). The elastic zone lies to the left of the 45º line, while the inelastic zone lies to the right of the 45º line in Figure 10. Along the 45º line itself, $|E_{DP}| = 1$.

Violation by Marshall's Constant Outlay Curve

Unitary elastic *DC* implies constancy of outlay, but the converse is not true. This means that constant outlay curve may give rise to elastic, inelastic, or unitary elastic *DC*. If this is so, $d(|E_{DP}|)/dD < 0$ is also violated. For example, if the constant outlay curve is represented by $E = AP^aD^b$, which is analogous to the Cobb-Douglas production function, where E constant expenditure or outlay, the *RDC* can be written as $y = Bx$, where $y = (P/D)$, $x = |dP/dD|$ and $B = (a/b)$. Hence, $y/x = ARDC = dy/dx = MRDC = |E_{DP}| = B$ and $E_{yx} = (dy/dx)/(y/x) = MRDC / ARDC = 1$. Thus, $|E_{DP}| \gtreqless 1$ is consistent with $E_{yx} = 1$ in the case of Cobb-Douglas type of constant outlay curve. It is noteworthy that $|E_{DP}| \gtreqless 1$ is only possible if $B \gtreqless 1$. Further, $d(|E_{DP}|)/dD = 0$ despite $E_{yx} = 1$ and irrespective of the value of $|E_{DP}|$ for all the three linear *RDCs*. All these results are shown in the Figure 11.

Conclusion

Most of the concave DCs obey the negative relationship between *APPED* and *D*. Almost all forms of convex *DC* do not obey such negative relationship, while very few forms of linear *DC* obey the negative relationship between *APPED* and *D*. The forms of *DC*, which do not support the traditionally established negative relationship between *APPED* and *D*, are abnormal, supernormal, or extraordinary. Further, unitary elastic *DC* implies constancy of outlay, but the converse is not true. This means that constant outlay curve may give rise to elastic, inelastic, or unitary elastic *DC*. If this is so, the negative relationship between *APPED* and *D* is violated.

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